Multi-Subject CST Mathematics Preparation

Competency 0003- 3.1 Geometry

Feb 2017
Agenda

Introduction to Competency

Content Review and Sample Problems

Additional Practice
Performance Expectations

The New York State Grade 7-12 Multi-Subject teacher

- Demonstrates knowledge of geometry as a formal mathematical system that is based on precise definitions, careful reasoning, and proof.
- Applies concepts of geometric transformations, congruence, symmetry, and similarity to prove geometric theorems.
- Uses triangle trigonometry and expresses geometric properties with equations.
- Uses measurement and geometry to model situations.
- Demonstrates knowledge of fundamental statistical concepts and their applications, works with and interprets data, uses measures of center and variability, and draws inferences from data distributions.
- Demonstrates knowledge of concepts associated with random sampling and random processes and methods for drawing valid conclusions.
- Calculates probabilities and understands how probability and statistics can be used to make decisions.
3.1 Geometry

- Analyzes transformation in the plane and congruence in terms of rigid motions
- Proves theorems about lines and angles, triangles, and parallelograms
- Analyzes formal geometric constructions (e.g., copying an angle, finding the perpendicular bisector of a segment) and methods for constructing figures (e.g., equilateral triangle, regular hexagon inscribed in a circle)
- Analyzes similarity in terms of similarity transformations and scale factors, and solves problems and proves theorems involving similar figures
- Defines trigonometric ratios, solves mathematical and real-world problems involving right triangles, and applies trigonometry to general triangles
- Analyzes and applies theorems about circles and finds lengths (e.g., diameters, radii, arcs, chords) and area of sectors of circles
- Expresses geometric properties with equations
3.1 Geometry

• Uses coordinate methods to prove geometric theorems algebraically
• Solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume, including problems involving cones, cylinders, and spheres
• Analyzes the shapes of two-dimensional cross sections of three-dimensional objects and identifies the three-dimensional objects generated by two-dimensional rotations
• Demonstrates knowledge of how to analyze and interpret assessment data to inform and plan instruction that engages and challenges all students to meet or exceed the NYCCLS related to geometry
Agenda

- Introduction to Competency
- Content Review and Sample Problems
- Additional Practice
Geometry: Transformation in the plane and congruence in terms of rigid motions

A **rigid motion** is the action of taking an object and moving it to a different location without altering its shape or size. Reflections, rotations, translations, and glide reflections are all examples of **rigid motions**.

**Reflection** – a transformation that flips a figure across a fixed point or line (of reflection) changing orientation but preserving shaping and size.

**Translation** – a transformation that slides a figure from one place to another preserving orientation, shape and size.

**Rotation** – a transformation that turns an figure around a point changing orientation but preserving shape and size.

**Glide Reflection** – a transformation consisting of a translation combined with a reflection about a plane parallel to the direction of the translation.

http://www.regentsprep.org/regents/math/geometry/math-GEOMETRY.htm#m1,
http://www.mathsisfun.com/geometry/transformations.html
Geometry: Transformation in the plane and congruence in terms of rigid motions

**Congruent (≅)** – geometric figures that are exactly the same in size and shape. Corresponding angles are the same, and corresponding sides are the same.

![Diagram of congruent triangles](image)

The corresponding congruent sides are marked with small straight line segments called hash marks. The corresponding congruent angles are marked with arcs. The order of the letters in the names of the triangles should display the corresponding relationships. By doing so, even without a picture, you would know that ∠A would be congruent to ∠D, and would be congruent to , because they are in the same position in each triangle name.

\[
\begin{align*}
BC & \cong EF \\
AB & \cong DE \\
AC & \cong DF
\end{align*}
\]
\[
\begin{align*}
\angle A & \cong \angle D \\
\angle B & \cong \angle E \\
\angle C & \cong \angle F
\end{align*}
\]

Common Misunderstanding: Geometric figures that change in size (dilate) are not congruent and are referred to as **Similar (∼)** figures.

http://www.regentsprep.org/regents/math/geometry/gp4/triangles.htm
Geometry: Transformation in the plane and congruence in terms of rigid motions- Translations

How do you complete rigid motions?

Translations \( T_{a,b}(x, y) = (x + a, y + b) \)

Move the figure left, right, up, and/or down according to the rule given. Example: \( T_{-7,-3}(x,y) = (x - 7, y - 3) \)

http://www.regentsprep.org/regents/math/geometry/math-GEOMETRY.htm#m1,
http://www.mathsisfun.com/geometry/transformations.html
Geometry: Transformation in the plane and congruence in terms of rigid motions - Reflections

How do you complete rigid motions?

**Reflections** \( r_k(\triangle ABC) = \triangle A'B'C' \). Reflect the same distance from the line or point of reflection as the corresponding point of the original figure.

http://www.regentsprep.org/regents/math/geometry/math-GEOMETRY.htm#m1, http://www.mathsisfun.com/geometry/transformations.html
Geometry: Transformation in the plane and congruence in terms of rigid motions- Rotations and Glide Reflections

How do you complete rigid motions?

**Rotations**  $R_{P,\theta}(A) = A'$.

**Watch this video.**

**Glide Reflections:** See below for an example. In this problem, the original triangle (ABC) was first reflected over the x axis (A’B’C’) and then translated to the left five units (A”B”C”)

http://www.regentsprep.org/regents/math/geometry/math-GEOMETRY.htm#m1,
http://www.mathsisfun.com/geometry/transformations.html
Geometry: Transformation in the plane and congruence in terms of rigid motions

Put it all together! Complete the sample problems below to determine congruency based on rigid motions:

Congruence with Rigid Motions
Properties of Congruent Figures

Additional Practice and Videos:
Translations
Rotations
Reflections
Reference Video
Use this reference to review common theorems, postulates, and properties associated with proofs.

On the next few pages, you will find a few good ones to know:

<table>
<thead>
<tr>
<th>Reflexive Property</th>
<th>A quantity is congruent (equal) to itself. ( a = a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric Property</td>
<td>If ( a = b ), then ( b = a ).</td>
</tr>
<tr>
<td>Transitive Property</td>
<td>If ( a = b ) and ( b = c ), then ( a = c ).</td>
</tr>
</tbody>
</table>
Geometry: Proves theorems about lines and angles, triangles, and parallelograms- Common Theorems, Postulates, Definitions, and Properties

Quadrilaterals
### Various Quadrilaterals and their Properties

| Parallelograms | About Sides | * If a quadrilateral is a parallelogram, the opposite sides are parallel.  
* If a quadrilateral is a parallelogram, the opposite sides are congruent. |
|----------------|-------------|---------------------------------------------------------------------|
|                | About Angles| * If a quadrilateral is a parallelogram, the opposite angles are congruent.  
* If a quadrilateral is a parallelogram, the consecutive angles are supplementary. |
|                | About Diagonals | * If a quadrilateral is a parallelogram, the diagonals bisect each other.  
* If a quadrilateral is a parallelogram, the diagonals form two congruent triangles. |

**Parallelogram**

If one pair of sides of a quadrilateral is BOTH parallel and congruent, the quadrilateral is a parallelogram.

**Rectangle**

If a parallelogram has one right angle it is a rectangle

A parallelogram is a rectangle if and only if its diagonals are congruent.

A rectangle is a parallelogram with four right angles.

**Rhombus**

A rhombus is a parallelogram with four congruent sides.

If a parallelogram has two consecutive sides congruent, it is a rhombus.

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

A parallelogram is a rhombus if and only if the diagonals are perpendicular.

**Square**

A square is a parallelogram with four congruent sides and four right angles.

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

**Trapezoid**

A trapezoid is a quadrilateral with exactly one pair of parallel sides.

**Isosceles Trapezoid**

An isosceles trapezoid is a trapezoid with congruent legs.

A trapezoid is isosceles if and only if the base angles are congruent.

A trapezoid is isosceles if and only if the diagonals are congruent.

A trapezoid is isosceles, the opposite angles are supplementary.

[http://www.regentsprep.org/regents/math/geometry/gpb/theorems.htm](http://www.regentsprep.org/regents/math/geometry/gpb/theorems.htm)
Geometry: Proves theorems about lines and angles, triangles, and parallelograms- Common Theorems, Postulates, Definitions, and Properties

- **Polygon** – a closed figure whose sides are line segments.

- **Regular Polygon** – a polygon with all sides congruent and all angles congruent.

- Polygons are named (classified) by their number of sides.
  - 3 sides – triangle
  - 4 sides – quadrilateral
  - 5 sides – pentagon
  - 6 sides – hexagon
  - 7 sides – heptagon
  - 8 sides – octagon
  - 9 sides – nonagon
  - 10 sides – decagon
  - n sides – n-gon

- For examples problems involving Polygons, watch the following video: [Polygons](#)
Geometry: Proves theorems about lines and angles, triangles, and parallelograms - Common Theorems, Postulates, Definitions, and Properties

Key terms:

- **Line** - set of points extending in two directions.
- **Ray** – part of a line extending in one direction with one endpoint
- **Line Segment** – part of a line with two end points
  - A line segment can be measured; lines and rays cannot
- **Angle** – formed by two rays originating from the same point, called the vertex.
- **Acute Angle** – measures between 0-90 degrees.
- **Right Angle** – measures exactly 90 degrees.
- **Obtuse Angle** – measures between 90-180 degrees.
- **Plane** – a flat surface.
Key terms:

- **Parallel Lines** - Lines that never intersect

- **Perpendicular Lines** - Lines that meet at a 90 degree angle.

- **Bisector** - Cuts a line, angle, etc. into equal parts.

- **Equidistant** - When points, lines, etc. are an equal distance away from one another.

- **Midpoint** - A point on the line segment that divides the line into two equal parts.

- **Equilateral Triangle** - All equal angles and sides

- **Scalene Triangle** - No sides or angles are equal

- **Isosceles Triangle** - The two sides/angles equal
Geometry: Proves theorems about lines and angles, triangles, and parallelograms- Common Theorems, Postulates, Definitions, and Properties

- **Intersecting** lines cross at a point.
- **Parallel** lines never cross.
- **Perpendicular** lines cross to form right angles.
- A **transversal** is a straight line that intersects two or more given lines at unique points.
- When two parallel lines are cut by a transversal eight angles are formed:

  ![Diagram of intersecting lines]

  - If two angles sum to 90 degrees, they are said to be **Complementary Angles**
  - If two angles sum to 180 degrees, they are said to be **Supplementary Angles**
Geometry: Proves theorems about lines and angles, triangles, and parallelograms - Common Theorems, Postulates, Definitions, and Properties

Corresponding, Alternate Interior, Alternate Exterior, and Vertical angles are congruent in measure. Linear pair angles and consecutive interior angles are supplementary (sum to 180)
Your turn! Put the ideas regarding angle relationships to test. Try out the following problems:

Check your understanding of Angle Relationships
Geometry: Proves theorems about lines and angles, triangles, and parallelograms- Common Theorems, Postulates, Definitions, and Properties

- **Right Triangle** – a triangle with one right angle.
- Know the Pythagorean Theorem! Guarantee you will use it at least once during your exam!

![Diagram of a right triangle with labels](image)

**Pythagorean Theorem:**
\[ a^2 + b^2 = c^2 \]

**Reverse:**
\[ a^2 = c^2 - b^2 \]
\[ b^2 = c^2 - a^2 \]

- For worked examples, check out these videos:
  - [The Pythagorean Theorem 1](#)
  - [The Pythagorean Theorem 2](#)
- Your Turn! Try out these problems involving Pythagorean Theorem.
Geometry: Proves theorems about lines and angles, triangles, and parallelograms- Common Theorems, Postulates, Definitions, and Properties

• **The Triangle Inequality Theorem:** the sum of the measures of any two sides of a triangle must be greater than the measure of the third side.
  - For example: you cannot have a triangle with side lengths 10, 11 and 22.

• If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
  - This is a key property and very useful

• The segment between the midpoints of two sides of a triangle is parallel to the third side and half as long.
  - Another key property that can be used in proofs and Geometry application problems
Geometry: Proves theorems about lines and angles, triangles, and parallelograms - Common Theorems, Postulates, Definitions, and Properties

There are five ways to determine if two triangles are congruent:

- **Side-Side-Side Congruence (SSS):** all three sides of both triangles are congruent

- **Side-Angle-Side Congruence (SAS):** two sides and the angle between the two sides (the included angle) of both triangles are congruent

- **Angle-Side-Angle (ASA):** two angles and the side in between them (the included side) of both triangles are congruent

- **Angle-Angle-Side (AAS):** two angles and a non-included side are congruent in both triangles

- **Hypotenuse-Leg (HL):** only applies to right triangles! The hypotenuse and a leg are congruent in both triangles.

Common Misunderstandings: AAA and SSA are not triangle congruency properties
Watch the following video on the triangle congruency postulates to learn more about each one:

[Link to Triangle Congruency Properties]

Practice with the Postulates

[Link to Check your understanding of the Triangle Congruency Postulates]
Geometry: Proves theorems about lines and angles, triangles, and parallelograms

Your turn! Try out the following tasks associated with proving theorems about lines, angles, triangles, and parallelograms:

- Use the Pythagorean Theorem to find the area of an equilateral triangle
- Prove Angles are Congruent
- Prove and apply relationships between the angles of a parallelogram by using congruent triangles
Geometry: Analyzes formal geometric constructions

Below you will find directions and visuals related to various constructions:

- How to copy an angle:
  Directions

- How to bisect an angle or a line segment:
  Directions

- How to construct a perpendicular line and perpendicular bisector:
  Directions

http://www.regentsprep.org/regents/math/geometry/gc1/perp.htm
Geometry: Analyzes formal geometric constructions

Below you will find directions and visuals related to various constructions:

- Equilateral Triangles:
  Directions

- Regular Hexagon inscribed in a circle (remember a regular figure is one that has all equal sides and equal angles)
  Directions

Check your understanding with these sample problems!

http://www.regentsprep.org/regents/math/geometry/gc1/perp.htm
Geometry: Analyzes similarity in terms of similarity transformations and scale factors, and solves problems and proves theorems involving similar figures

Figures are similar if their corresponding angles are congruent and their corresponding sides have the same ratio.

Below you will find an example of how to determine if two figures are similar from a given set of transformations:

- $180^\circ$ rotation about point C or a vertical and horizontal reflection over perpendicular lines that intersect at point C.
- A dilation with center at C and scale factor $\frac{3}{2}$ to map CED onto CAB or $\frac{2}{3}$ to map CAB onto CED.
Geometry: Analyzes similarity in terms of similarity transformations and scale factors, and solves problems and proves theorems involving similar figures

The following video describes how to solve problems involving similar figures:

Determine a scale factor by applying ratios
Additional Resource

Test your knowledge! Try the following problems involving similarity:

Transformations and Similarity
Similarity Triangle Problems
Geometry: Trig Ratios

Let’s begin with right triangle ABC,

θ = “theta” = angle of perspective
A = measure of angle A
B = measure of angle B
C = measure of angle C = 90°

NOTE: The sum of all angles in a triangle always equals 180°.

a = side opposite angle A
b = side adjacent to angle A
c = side opposite the right angle = hypotenuse
Geometry: Trig Ratios

The basic formulas relative to angle A in right triangle ABC are as follows:

- **Sine of** \( \angle A = \sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c} \)
- **Cosine of** \( \angle A = \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c} \)
- **Tangent of** \( \angle A = \tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b} \)
- **Cosecant of** \( \angle A = \csc A = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{c}{a} \)
- **Secant of** \( \angle A = \sec A = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{c}{b} \)
- **Cotangent of** \( \angle A = \cot A = \frac{\text{adjacent}}{\text{opposite}} = \frac{b}{a} \)

**Notes:**
- Sine and cosecant are reciprocals
- Cosine and secant are reciprocals
- Tangent and cotangent are reciprocals
Geometry: Trig Ratios

More on the Trig functions: The six basic trigonometric ratios
Geometry: Trig Ratios

Calculators default to radian measures. Be sure to change your MODE to degrees when looking for and using angles given in degrees!
Geometry: Defines trigonometric ratios, solves mathematical and real-world problems involving right triangles, and applies trigonometry to general triangles

Trig ratios can be used to solve for missing side lengths of right triangles. Try out a few problems, using SOH-CAH-TOA

- From the top of a barn 25 feet tall, you see a cat on the ground. The angle of depression of the cat is 40º. How many feet, to the nearest foot, must the cat walk to reach the barn?

- A ladder leans against a building. The foot of the ladder is 6 feet from the building. The ladder reaches a height of 14 feet on the building. Find the length of the ladder to the nearest foot.

Additional Problems and Solutions to above problems

http://www.regentsprep.org/regents/math/algebra/AT2/PracTrig.htm
Geometry: Defines trigonometric ratios, solves mathematical and real-world problems involving right triangles, and applies trigonometry to general triangles

Inverse Trig Functions can be used to solve for angles of right triangles. Make sure your calculator’s mode is set to degree!

Watch this video (Good example of application)

Then give this problem a try! Solution

Let’s Share

What side lengths do you know?

Which trigonometric function uses those sides?

How do you write the equation using the inverse of that trigonometric function to solve for $x$?
Geometry: Defines trigonometric ratios, solves mathematical and real-world problems involving right triangles, and applies trigonometry to general triangles

Below you will find information regarding the Law of Sines and the Law of Cosines, which allow you to find missing side and angle lengths of non-right triangles.

Video on Law of Sines

Ambiguous Case

Video on Law of Cosines

Additional Problems on Law of Cosines

To differentiate between when to use Law of Sines and Law of Cosines remember that the Law of Cosines can be used for triangles that include all three side lengths or two sides and an included angle (SAS).
Geometry: Defines trigonometric ratios, solves mathematical and real-world problems involving right triangles, and applies trigonometry to general triangles

Put it all together! Check your understanding of each rule and then when to use which one.

- Practice Law of Sines
- Practice Law of Cosines
- Practice Right Triangle Trig
- Apply trig in contextual problems
Geometry: Analyzes and applies theorems about circles and finds lengths (e.g., diameters, radii, arcs, chords) and area of sectors of circles

- Key definitions with circles:
  - **Circle** – set of all coplanar points equidistant from the same point, called the center.
  - **Radius** – the distance from the center to any point on the circle.
  - **Diameter** – the distance from point on the circle to another point through the center.
    - 1 diameter = 2 radii
  - **Circumference** = distance around the circle
    - $C = 2\pi r$ or $C = \pi d$
  - **Area** of a circle, $A = \pi r^2$
  - **Concentric circles** – have the same center
  - **Arc** – part of a circle
  - **Semicircle** – an arc with endpoints of a diameter.
Geometry: Analyzes and applies theorems about circles and finds lengths (e.g., diameters, radii, arcs, chords) and area of sectors of circles

- **Chord** – a line segment with end points on the circle.

- **Secant** – a line that passes through a circle

- **Tangent** – a line that intersects the circle in only one point, the point of tangency.
  - A tangent line is perpendicular to the radius at the point of tangency.

- **Central angle** – an angle whose vertex is the center of a circle.
  - **Minor arc** – an arc of measure 0-180 degrees.
  - **Semicircle** – an arc of 180 degrees
  - **Major arc** – an arc of measure 180-360 degrees.

- **Inscribed angle** – an angle whose vertex is on the circle.

https://www.mathsisfun.com/geometry/circle.html
More fun stuff with circles...

- An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle.
- The measure of an inscribed angle is half the measure of its intercepted arc.
- The length of an arc is: \( s = r \theta \), \( \theta \) in radians and \( r \) is the radius.

In this set of lessons, you will take an in depth look at identifying and describing relationships among inscribed angles, radii, and chords.

These two resources also outline theorems associated with finding the lengths of chords.

- **Circles and Chords**
- **Segment Rules in Circles**
Geometry: Analyzes and applies theorems about circles and finds lengths (e.g., diameters, radii, arcs, chords) and area of sectors of circles

Below are various formulas you can use to find the area of sectors and arc lengths:

- **Area of Sectors:** \( A = \frac{\theta}{360} \pi r^2 \) where \( \theta \) = the central angle given in degrees or \( A = \frac{\theta}{2\pi} \pi r^2 \) (not simplified) where \( \theta \) = the central angle given in radians.

  The area of a circle is \( A = \pi r^2 \). Therefore, if we are looking for the area of a “section” given by the central angle take the formula for the area of the circle times the fraction of the circle selected.

- **Length of an Arc:** \( L = \frac{\theta}{360} \pi d \) where \( \theta \) = central angle given in degrees or \( L = \frac{\theta}{2\pi} \pi d \) (not simplified) where \( \theta \) = central angle given in radians.

  The circumference of a circle is \( C = \pi d \). Therefore, if we are looking for the length of a “section” of the arc, we can take the formula for the circumference times the fraction of the circle selected.

Note: Some formulas above were not simplified in an effort to make them easier to remember.
Geometry: Analyzes and applies theorems about circles and finds lengths (e.g., diameters, radii, arcs, chords) and area of sectors of circles

Put your knowledge of circles to the test! Try out the practice problems below!

- Area of Sectors and Arc Length
- Chords
- Angle Relationships
- Diameter and Radii
Geometry: Expresses geometric properties with equations

**Parallel and Perpendicular Lines**—this information will also prove useful in the next section on using coordinate methods to prove geometric theorems

Parallel lines have the same slope
Example: Create an equation of a line
Parallel to \( y = 2x + 1 \) containing the point (5,4)

\[
Y = 2x + b \\
4 = 2(5) + b \\
-6 = b \\
Y = 2x - 6
\]

Perpendicular lines have negative reciprocal Slope. Example: Create an equation of a line
Perpendicular to \( y = 2x + 1 \) containing the point (0,4).

\[
Y = -\frac{1}{2}x + b \\
4 = -\frac{1}{2}(0) + b \\
b = 4 \\
Y = -\frac{1}{2}x + 4
\]

https://www.mathsisfun.com/algebra/line-parallel-perpendicular.html
Look below to learn how to use the Pythagorean Theorem to derive the Distance Formula:

Using the Pythagorean Theorem: \( a^2 + b^2 = c^2 \), we can find the distance between points \((x_1, y_1)\) and \((x_2, y_2)\)

\[
\begin{align*}
   a &= |x_1 - x_2| \quad b = |y_1 - y_2| \quad c = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}
\end{align*}
\]
Below are formulas and videos for some conic sections. Definitely know the equation for a circle

- **Standard Equation of a Circle** (watch this video on deriving this equation from the distance formula)
  \[(x - h)^2 + (y - k)^2 = r^2\]
  
  \((h, k)\) center, \(r\) radius

- Here are some practice problems of having to complete the square and write a circle in standard form:
  - **Equation of Circle Practice**
The following videos describe the other conic sections:

- **Ellipses**

- **Parabolas**

- **Hyperbolas**

 Geometry: Expresses geometric properties with equations
Geometry: Expresses geometric properties with equations

Use the following practice sets to assess your understanding of conic sections:

1. Ellipses
2. Parabolas
3. Hyperbolas
Geometry: Uses coordinate methods to prove geometric theorems algebraically

Watch the following videos to learn how to use coordinate methods to prove geometric theorems:

1. **Prove whether a figure is a rectangle on the coordinate plane**
2. **Prove whether a point is on a circle**

Good formulas to know include:

- **Distance Formula:** \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
- **Mid-point Formula:** \[ (x, y) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]
- **Slope Formula:** \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
Geometry: Uses coordinate methods to prove geometric theorems algebraically

Test your knowledge! Try out the following applications:

- Prove a figure is a rectangle
- Classifying Quadrilaterals
- Area and Perimeter application
- Additional Practice
Geometry: Solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume, including problems involving cones, cylinders, and spheres

- **Triangles**
  \[ A = \frac{1}{2} b \times h \]
  \[ P = s_1 + s_2 + s_3 \]
  
  \[ A = \text{Area} \]
  \[ P = \text{Perimeter} \]
  \[ h = \text{height} \]
  \[ b = \text{base} \]
  \[ l = \text{length} \]
  \[ w = \text{width} \]

- **Squares**
  \[ A = s^2 \]
  \[ P = 4s \]

- **Rectangles**
  \[ A = lw \]
  \[ P = 2l + 2w \]

- **Parallelograms**
  \[ A = b \times h \]
  \[ P = 2a + 2b \]
**Geometry:** Solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume, including problems involving cones, cylinders, and spheres

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula / Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Circle:</strong> radius ( r ), diameter ( d )</td>
<td>area = ( \pi r^2 )</td>
</tr>
<tr>
<td></td>
<td>circumference = ( 2\pi r - \pi d )</td>
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<tr>
<td></td>
<td>diameter ( d = 2r )</td>
</tr>
<tr>
<td><strong>Trapezoid:</strong> height ( h ), bases ( a, b )</td>
<td>area = ( \frac{1}{2}h(a + b) )</td>
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<td></td>
<td>perimeter = ( a + b + c + d )</td>
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<tr>
<td><strong>Sphere:</strong> radius ( r )</td>
<td>volume = ( \frac{4}{3}\pi r^3 )</td>
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<tr>
<td></td>
<td>lateral surface area = ( 4\pi r^2 )</td>
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<td></td>
<td>total surface area = ( 2\pi r^2 ) + sum of area of sides</td>
</tr>
<tr>
<td><strong>Right prism:</strong> height ( h ), area of base ( B )</td>
<td>volume = ( Bh )</td>
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<tr>
<td></td>
<td>total surface area = ( 2B + \text{sum of area of sides} )</td>
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<tr>
<td><strong>Rectangular prism:</strong> length ( l ), width ( w ), height ( h )</td>
<td>volume = ( lwh )</td>
</tr>
<tr>
<td></td>
<td>total surface area = ( 2lh + 2hw + 2lw )</td>
</tr>
<tr>
<td><strong>Cube:</strong> side ( s )</td>
<td>volume = ( s^3 )</td>
</tr>
<tr>
<td></td>
<td>total surface area = ( 6s^2 )</td>
</tr>
<tr>
<td><strong>Right circular cylinder:</strong> height ( h ), radius of base ( r )</td>
<td>volume = ( \pi r^2 h )</td>
</tr>
<tr>
<td></td>
<td>lateral surface area = ( 2\pi rh )</td>
</tr>
<tr>
<td></td>
<td>total surface area = ( 2\pi rh + 2\pi r^2 )</td>
</tr>
<tr>
<td><strong>Pyramid:</strong> height ( h ), area of base ( B )</td>
<td>volume = ( \frac{1}{3}Bh )</td>
</tr>
<tr>
<td><strong>Right circular cone:</strong> height ( h ), radius of base ( r )</td>
<td>volume = ( \frac{1}{3}\pi r^2 h )</td>
</tr>
<tr>
<td></td>
<td>lateral surface area = ( \pi r \sqrt{r^2 + h^2} - \pi rs ), where ( s ) is the slant height = ( \sqrt{r^2 + h^2} )</td>
</tr>
<tr>
<td></td>
<td>total surface area = ( \pi r \sqrt{r^2 + h^2} + \pi r^2 - \pi rs + \pi r^2 )</td>
</tr>
</tbody>
</table>
Geometry: Solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume, including problems involving cones, cylinders, and spheres

**Triangle:** sides $a$, $b$, and $c$
- Area: $\frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$
- Perimeter: $s = \frac{a + b + c}{2}$

**Equilateral triangle:** side $s$
- Area: $\frac{\sqrt{3}}{4}s^2$
- Perimeter: $3s$

**Isosceles triangle:** sides $a$, $a$, and $b$
- Area: $\frac{1}{2}b\sqrt{a^2 - \frac{b^2}{4}}$
- Perimeter: $2a + b$

**Sector of circle:** radius $r$, $\theta$
- Area: $\frac{\theta r^2}{2}$
- Arc length: $s = r\theta$
Geometry: Solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume, including problems involving cones, cylinders, and spheres

Below are links to further explanations and worked examples involving volume, perimeter, area, and surface area

- [Cylinder Volume and Surface Area](#)

- [Volume of a Sphere](#)

- [Perimeter, area, and volume](#) (Select the videos that are needed)
Geometry: Solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume, including problems involving cones, cylinders, and spheres.

On the next few slides you will find a few examples of real-world problems involving perimeter, area, surface area, and volume.

**Task**
Terrifics Tents made a new two person tent. If the material costs $0.04 per square in, determine how much it will cost to make the two person tent.

**Task Solution**

Area of each rectangle
length x width = (80 in)(50 in) = 4,000 in²

Area of each triangle
\[ \frac{1}{2}bh = \frac{1}{2}(50\text{ in})(40\text{ in}) = 1,000\text{ in}^2 \]

Area of net
3(4,000 in²) + 2(1,000 in²) = 14,000 in²

Total Cost
$0.04(14,000\text{ in}^2) = $560.00

https://learnzillion.com/lesson_plans/801#fndtn-lesson
Geometry: Solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume, including problems involving cones, cylinders, and spheres

Real-world example 2:

Ten dartboard targets are being painted as shown in the following figure. The radius of the smallest circle is 3 in. and each successive, larger circle is 3 in. more in radius than the circle before it. A “tester” can of red and of white paint is purchased to paint the target. Each 8 oz. can of paint covers 16 ft². Is there enough paint of each color to create all ten targets?

Let each circle be labeled as in the diagram.

Radius of $C_1$ is 3 in.; area of $C_1$ is $9\pi$ in².
Radius of $C_2$ is 6 in.; area of $C_2$ is $36\pi$ in².
Radius of $C_3$ is 9 in.; area of $C_3$ is $81\pi$ in².
Radius of $C_4$ is 12 in.; area of $C_4$ is $144\pi$ in².
Geometry: Solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume, including problems involving cones, cylinders, and spheres

Solution:

Target area painted red

The area between $C_4$ and $C_3$: $144\pi \text{ in}^2 - 81\pi \text{ in}^2 = 63\pi \text{ in}^2$

The area between $C_2$ and $C_1$: $36\pi \text{ in}^2 - 9\pi \text{ in}^2 = 27\pi \text{ in}^2$

Area painted red in one target: $63\pi \text{ in}^2 + 27\pi \text{ in}^2 = 90\pi \text{ in}^2$; approximately $282.7 \text{ in}^2$

Area of red paint for one target in sq. ft.: $282.7 \text{ in}^2 \left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right) \approx 1.96 \text{ ft}^2$

Area to be painted red for ten targets in sq. ft.: $1.96 \text{ ft}^2 \times 10 = 19.6 \text{ ft}^2$

Target area painted white

The area between $C_3$ and $C_2$: $81\pi \text{ in}^2 - 36\pi \text{ in}^2 = 45\pi \text{ in}^2$

The area of $C_1$: $9\pi \text{ in}^2$

Area painted white in one target: $45\pi \text{ in}^2 + 9\pi \text{ in}^2 = 54\pi \text{ in}^2$; approximately $169.6 \text{ in}^2$

Area of white paint for one target sq. ft.: $169.6 \text{ in}^2 \left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right) \approx 1.18 \text{ ft}^2$

Area of white paint needed for ten targets in sq. ft.: $1.18 \text{ ft}^2 \times 10 = 11.8 \text{ ft}^2$

There is not enough red paint in one tester can of paint to complete all ten targets; however, there is enough white paint in one tester can of paint for all ten targets.
Geometry: Solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume, including problems involving cones, cylinders, and spheres

Real-world example 3:

The box below measures 12 inches high, 15 inches long, and 8 inches wide. A typical soda can measures 4.8 inches high and 2.5 inches in diameter. If the cans must sit upright, how many cans can you fit into the box and how much wasted space will be in the box?

Task Solution:
12 in/4.8 in = 2.5, so 2 layers of cans are possible
15 in/2.5 in = 6, so 6 cans along the length
8 in/2.5 in = 3.2, so 3 cans along the width; therefore 2(6)(3) = 36 cans
Volume of box: $V = 12\text{in}(15\text{in})(8\text{in}) = 1440 \text{in}^3$
Volume of 36 cans: $36(3.14)(1.25\text{in})^2(4.8\text{in}) = 847.8 \text{in}^3$
Wasted Space: $1440 \text{in}^3 - 847.8 \text{in}^3 = 592.2 \text{in}^3$ (or about 41% of the box)
Geometry: Solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume, including problems involving cones, cylinders, and spheres

Test your knowledge! Try the following problems!

- Which Deal is the Best? Using Volume
- Coordinate Plane: Equal Area Triangles
- Real Life Perimeter and Area Problems (includes some good problems with solutions on teacher lesson plan)
Geometry: Analyzes the shapes of two-dimensional cross sections of three-dimensional objects and identifies the three-dimensional objects generated by two-dimensional rotations

Watch the following video set to learn how to analyze the shapes of two dimensional cross sections of three-dimensional objects and identify the three-dimensional objects generated by two dimensional rotations.

Cross Sections

Check your understanding:
1. Comparing Cross Sections
2. Application of Cross Section

https://learnzillion.com/lessonsets/505
Agenda

Introduction to Competency

Content Review and Sample Problems

Additional Practice
ABCD is a rectangle in the coordinate plane. If the coordinates of point A are (-2, 1) and the coordinates of point C are (6, 3), which of the following are possible coordinates of points B and D?

A. (-3, 2) and (1, 6)
B. (3, -2) and (1, -6)
C. (-2, 3) and (6, 1)
D. (2, -3) and (-6, -1)
Solution: C

A figure formed by connecting the two given points and the two unknown points must be a rectangle, i.e., a quadrilateral in which the pairs of opposite sides are congruent and parallel, and whose adjacent sides intersect at right angles. Draw a set of coordinate axes and plot the two given points and the pairs of points in each response. Only the points in response C result in a figure meeting the specified criteria.
Use the diagram below to answer the question that follows:

Two posts, one 12 feet high and the other 22 feet high, stand 20 feet apart. They are held in place by two wires, attached to a single stake, running from ground level to the top of each post. If \( x \) represents the distance from the base of the 12-foot post to the stake, which of the following expressions represents the total length of the wire in terms of \( x \)?

A. \( \sqrt{x^2 + 144} + \sqrt{x^2 + 484} \)
B. \( \sqrt{x^2 + 144} + \sqrt{x^2 - 60x + 1384} \)
C. \( \sqrt{x^2 + 24x + 144} + \sqrt{x^2 + 44x + 484} \)
D. \( \sqrt{x^2 + 24x + 144} + \sqrt{x^2 - 60x + 1384} \)
Solution: B

The Pythagorean theorem can be used to calculate the length of each portion of the wire. Let $l$ represent the length of the wire from the 12-foot post to the stake. Since $x^2 + (12)^2 = l^2$, then $\sqrt{x^2 + 144} = l$. Let $r$ represent the length of the wire from the 22-foot post to the stake. The distance from the stake to the 22-foot post is $30 - x$. Since $(30 - x)^2 + (22)^2 = r^2$, then $\sqrt{(30 - x)^2 + 484} = \sqrt{x^2 - 60x + 1384} = r$. Therefore, the total length $l + r$, of the wire is $\sqrt{x^2 + 144} + \sqrt{x^2 - 60x + 1384}$. 
Use the figure below to answer the question that follows.

Two arcs of the same radius are drawn, one centered at point A and the other at point B. Segment DE is then drawn, connecting the points where the arcs intersect, and intersecting segment AB at point F. Segment FC is then drawn from vertex C to point F. Which of the following best describes segment FC?

A. A perpendicular bisector
B. A median
C. An angle bisector
D. An altitude
Solution: B

The procedure described to produce segment $DE$ is the compass-and-straightedge construction of the perpendicular bisector of line segment $AB$. Hence, $AF = BF$ and $F$ is the midpoint of segment $AB$. By definition, a median of a triangle is a segment from a vertex to the midpoint of the opposite side. Therefore, segment $FC$ is a median.
A flashlight has a circular face and is tilted at a non-zero acute angle, \( \theta \), to create a closed illuminated figure on a vertical wall as shown above. Which of the following equations could be used to represent the border of the figure if an \( x \)-\( y \) coordinate system is placed on the wall?

A. \( \frac{x^2}{a^2} + \frac{y}{b} = 1 \)
B. \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \)
C. \( \frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \)
D. \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \)
Solution: B

The flashlight creates a cone of light, and the intersection of the wall (a plane) and the cone is a conic section. Since $\theta$ is an acute angle, the conic section is an ellipse. Placing a coordinate system on the wall with the ellipse centered at the origin results in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which is the equation of an ellipse in standard form.
Use the diagram below to answer the question that follows.

The transformation $T$ of $\triangle ABC$ to $\triangle A'B'C'$ is a dilution with center at $O$. If the area of $\triangle A'B'C'$ is twice the area of $\triangle ABC$, what is the scale factor of $T$?

A. $\frac{1}{2}$  
B. $\frac{\sqrt{2}}{2}$  
C. $\sqrt{2}$  
D. 2
**Solution:** C

A dilution is a similarity transformation. Therefore, $\triangle ABC \sim \triangle A'B'C'$. If the dilation has scale factor $k$, then the area of $\triangle A'B'C' = k^2 \times (\text{the area of } \triangle ABC) = 2 \times (\text{the area of } \triangle ABC)$. Therefore, $k^2 = 2$ and $k^2 = \sqrt{2}$. 
Use the diagram below to answer the question that follows.

A target consists of a square region in which a quarter circle is drawn and shaded. The radius of the circle is equal to one side of the square. A computer program calculates pi (π) by simulating darts being fired randomly at the target. A success is defined as a dart falling within the shaded area. If the computer obtains a value of 3.12 after 1000 shots at the target, how many darts landed in the shaded area? Assume that the probability of hitting the target is one.

A. 312  
B. 624  
C. 750  
D. 780
Solution: D

Let N represent the number of darts that land in the shaded area. The relative frequency of the landing darts is \( f = \frac{N}{1000} \). The probability that a randomly fired dart lands in the quarter circle is equal to the ratio of the area of the quarter circle to the area of the square. Since the number of shots is relatively large, this probability is approximately equal to the relative frequency. Let \( s \) represent the length of the side of the square. Since the area of a circle is \( \pi s^2 \), we obtain \( \frac{N}{1000} = \frac{\left(\frac{\pi}{4}\right)s^2}{s^2} \). Substituting 3.12 for the value of the value obtained for \( \pi \) by the computer, simplifying and solving for \( N \) results in \( N = 780 \).
Which of the following sets of transformations will produce triangle A’B’C’ that is congruent to triangle ABC?

Triangle ABC is rotated 90° and translated down 3 units
Triangle ABC is dilated by a scale of 2 and rotated 270°
Triangle ABC is translated up 4 units then translated left 5 units

A. I and II
B. II and III
C. I, II, and III
D. I and III
E. III only
Solution: D

In order to maintain congruence, Triangle ABC must maintain the same scale factor. Rotations, reflections, and translations maintain congruence, dilations do not.
Calculate the area $A$ of triangle ABC in the figure below. The lengths of the sides $AB$ and $BC$ are $AB = 4$ and $BC = 5$.

A. $A = 3.5$
B. $A = 5$
C. $A = 6$
D. $A = 7.5$
Solution: B

Draw the altitude to angle A which will create a right triangle. The altitude will become the opposite side to angle 30 in the right triangle. To find the altitude (height) use the trig ratio for sine.

\[
\sin(\Theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}
\]

\[
\sin(30) = \frac{\text{Opposite}}{4}
\]

\[
4 \times \sin(30) = \text{Opposite}
\]

\[
2 = \text{Opposite} \quad \text{<--- this is the height of the triangle}
\]

Area = \(\frac{1}{2}\) base \times \text{height}

Area = \(\frac{1}{2}\) 5 \times 2