Multi-Subject CST Mathematics Preparation

Competency 0004- Analysis, Synthesis, and Application

February 2017
Agenda

Introduction to Competency

- Strategies and Scoring
- Example Analyzing Classroom Exhibits
- Practice Analyzing Classroom Exhibits
Introduction to Competency 0004- Analysis, Synthesis, and Application

Performance Expectations

The New York State Grade 7-12 Multi-Subject teacher accurately and effectively

• Applies relevant content knowledge and pedagogical content knowledge in number and operations, operations and algebraic thinking, fractions, ratio and proportional reasoning, and measurement and data to analyze and synthesize assessment data about an individual student, identify conceptual or procedural errors, and provide a well-reasoned and accurate analysis of a student's mathematical knowledge.

• Uses the assessment results and knowledge of how students learn to present an appropriate instructional approach that meets the needs of the student.
Performance Indicators

• Analyzes and interprets samples of a student's work and other assessment data to monitor student progress and determine a student's strengths and areas of need in Mathematics

• Demonstrates knowledge of the content by identifying and analyzing any errors or misconceptions in work samples

• Describes appropriate and effective content-specific instructional strategies, activities, or interventions to address a student's identified needs

• Demonstrates the ability to generate real-world scenarios that illustrate specific mathematical concepts

• Demonstrates the ability to justify the effectiveness of selected instructional strategies, activities, or interventions for promoting a student's mathematical understanding
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Introduction to Competency

**Strategies and Scoring**

Example Analyzing Classroom Exhibits

Practice Analyzing Classroom Exhibits
Strategy: Writing the constructed response

Goal: Identify a specific need of the student(s), share a strategy to address the student learning need, and explain why your plan will meet the need of the student(s).

• Identify a specific strength that the student demonstrates - pull evidence and language from the appropriate classroom exhibits that explain why you believe this is a strength.

• Identify a specific learning need that the student demonstrates based on any discrepancies between the student’s method and response and the correct response - pull evidence and language from the appropriate classroom exhibits that explain why you believe this is an area in need of development.

• Describe the intervention that you would use to build on the student’s strength and would help support the identified area of need.

• Explain why the strategy that you choose would be effective in supporting the student in building a viable argument that is related to the standard.

You should write your response to the question in approximately 50 min.
# Level 4 constructed response performance characteristics

<table>
<thead>
<tr>
<th>Performance Characteristic</th>
<th>Description</th>
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<tbody>
<tr>
<td>Completeness</td>
<td>The degree to which the response addresses all parts of the assignment</td>
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<tr>
<td>Accuracy</td>
<td>The degree to which the response demonstrates the relevant knowledge and skills accurately and effectively</td>
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<tr>
<td>Depth of Support</td>
<td>The degree to which the response provides appropriate examples and details that demonstrate sound reasoning</td>
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<table>
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<tr>
<th>Score Point</th>
<th>Score Point Description</th>
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| 4           | The "4" response reflects a thorough command of the relevant knowledge and skills:  
• The response thoroughly addresses all parts of the assignment.  
• The response demonstrates the relevant knowledge and skills with thorough accuracy and effectiveness.  
• The response is well supported by relevant examples and details and thoroughly demonstrates sound reasoning. |
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Strategies and Scoring

Example Analyzing Classroom Exhibits

Practice Analyzing Classroom Exhibits
Example analyzing classroom exhibits

Use the data provided to complete the task that follows.

Using the data provided, prepare a response of approximately 400–600 words in which you:

- Identify a significant mathematical strength related to the given standard that is demonstrated by the student, citing specific evidence from the exhibits to support your assessment;
- Identify a significant area of need related to the given standard that is demonstrated by the student, citing specific evidence from the exhibits to support your assessment; and
- Describe an instructional intervention that builds on the student's strengths and that would help the student improve in the identified area of need. Include a strategy for helping the student build a viable argument related to the given standard.
Example analyzing classroom exhibits: What skills and knowledge are encompassed within this standard?

A teacher is working with a student in a high-school algebra class. The class is currently working on the following standard from the New York State P–12 Common Core Learning Standards for Mathematics.

Reasoning with Equations & Inequalities (A-REI)
Solve equations and inequalities in one variable.

4b. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

In particular, the class is working on solving quadratic equations by factoring. Previous classes have covered the structure of expressions, writing expressions in equivalent form, and performing arithmetic with polynomials.
Example analyzing classroom exhibits: How do you solve these problems? What do the student’s responses tell us about their understanding of these math concepts?

Worksheet Directions: Solve for x. Check your work and justify the steps you used to find the solution.

Problem 1: \(x^2 + 2x - 3 = 0\)

\[x^2 + 2x - 3 = 0\]
factor this
\[(x \text{ )}(x \text{ })\]
look at \(-3\)
\[-3 \cdot 1 = -3 \quad 3 \cdot -1 = -3\]
\[3 + -1 = 2\]
\[(x + 3)(x - 1) = 0\]
solve each of these for \(x\)
\[(x + 3)\]
\[x + 3 = 0\]
\[-3 -3\]
x = -3
\[(x - 1)\]
\[(x - 1) = 0\]
x = +1
\[\text{check answer}\]
\[x = -3\]
x = \(-3\)
\[x^2 + 2x - 3 = 0\]
\[(x + 3)(x - 1) = x\cdot x + x\cdot -1\]
\[+ 3x + 3\cdot -1\]
\[x^2 - x + 3x - 3\]

You find the solution by solving the equation. Plug the answer back into the equation to check it.
Example analyzing classroom exhibits: What else do we learn about student understanding from their explanation of the math?

Shown below is an excerpt of an interview between the teacher and the student in which the teacher questions the student about the method of solution used to solve the problems shown in the student work sample.

**Teacher:** We will start with Problem 1. Can you tell me what you did to solve this equation?

**Student:** First I wrote down two sets of parentheses. Then I factored it by looking for two numbers that when multiplied give 3, no, minus 3, and when you add them you get 2. I wrote out possible answers and found that 3 and minus 1 worked. Then I put them in with the x's like here (points to the product of two binomials set equal to zero).

**Teacher:** It looks like after you factored the trinomial into two binomials, you set each of them equal to zero and solved the resulting equation. Can you tell me why you took that step? Can you justify why you set each binomial equal to zero?

**Student:** Whatever is on the left side of the equation has to equal what's on the right side. So $(x + 3)$ has to equal zero and so does $(x – 1)$, because equations have to balance. Then you just add or subtract the number to find the answer.

**Teacher:** Let’s look at Problem 2. Can you tell what strategy you used to solve the equation?

**Student:** I solved it just like I solved the first one. This time, though, it was easier to solve because it’s easier to factor. You just take out an $x$.

**Teacher:** After you factored the left side of the equation, what did you do next?

**Student:** I set both sides of the equation equal to 8 and solved for $x$.

**Teacher:** Can you justify why you took that step?

**Student:** Both sides of an equation must be equal. So whatever is on the left side has to equal what is on the right side. So I set them equal to each other and solved for $x$. 

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A significant strength demonstrated by the student is her understanding of how to factor a polynomial. When factoring Problem 1, the student knew two numbers were needed that "when multiplied give minus 3, and when you add them you get 2" as the constant terms in the binomials. Although she made many errors in Problem 2, she correctly factored $x^2 + 6x = -8$ as $(x)(x + 6) = 8$.

One significant need demonstrated by the student is her misunderstanding of the Zero Product Property. A zero product indicates that at least one of the factors must also equal zero. The student demonstrates an incomplete understanding of this property in the interview when she explains that both $(x + 3)$ and $(x - 1)$ have to equal zero "because equations have to balance." She then apparently generalizes this flawed understanding and incorrectly concludes that a product other than zero must have factors equal to that number, as shown in Problem 2: "$x = -8$" and "$x + 6 = -8$" and her explanation that "I set both sides of the equation equal to 8" because "both sides of an equation must be equal." The student may not know the reason for factoring, and is simply factoring the polynomials because the assignment was "solve several quadratic equations by factoring."

Instructional intervention should start with the Zero Product Property. This concept is pivotal in understanding how and why to use factoring as a solution method for quadratic equations. The teacher could ask her to identify a list of possible solutions for A and B in the problem $A * B = 0$. Then ask for a list of possible solutions for A and B in the problem $A * B = 12$. The student will then compare the lists. The teacher should question the student to help her understand the property: What does she notice? Which list is easier to create and why? What is special about the number 0 when you are finding factors?
Next, ask the student to solve a problem such as this: \((x + 8)(x - 2) = 0\). The student will probably solve this correctly, as she correctly solved \((x + 3)(x - 1) = 0\) in Problem 1. The teacher should draw her attention to the similarity in her approaches to this problem and the previous \(A \times B = 0\) problem.

Next, ask the student to solve a quadratic equation such as \(x^2 - 5x + 4 = 0\). The student would likely factor it correctly, as she has already shown her ability to factor. This time, as she set each factor to equal zero, the teacher would have her explain why she is doing that. Next, the student would be given an equation such as \(x^2 + 5x = -6\). Drawing the student's attention back to the original \(A \times B\) problems, the teacher would elicit the observation from the student that it was easier to solve the equation when the product was zero rather than another number because she knew that one of the factors would have to equal zero, and then guide the student to write an equivalent expression with a product of zero before solving.

Working from what she already knows (how to factor), the student will begin to build a viable argument for factoring as a solution method for quadratic equations as she comprehends the mathematical concept (the Zero Product Property) that underlies all the problems, from simple to more complex. To help her continue to build a viable argument, the teacher should give her a few more problems (some set up with the product set at zero, some with a product set as non-zero) and have her explain her steps and solutions.
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Introduction to Competency

Strategies and Scoring

Example Analyzing Classroom Exhibits

Practice Analyzing Classroom Exhibits 1
Practice analyzing classroom exhibits #1: What skills and knowledge are encompassed within this standard?

A teacher is working with students in a 7th grade class. The class is currently working on the following standards from the Common Core Learning Standards for Mathematics.

• **CCSS.MATH.CONTENT.7.RP.A.1**
  Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction \( \frac{1/2}{1/4} \) miles per hour, equivalently 2 miles per hour.*

• **CCSS.MATH.CONTENT.7.RP.A.2**
  Recognize and represent proportional relationships between quantities.

In particular, the class is working on identifying and connecting the constant of proportionality in a given situation to the corresponding equation. Previous classes have covered how to calculate unit rates, divide complex fractions, and use tables in real life situations.
Practice analyzing classroom exhibits #1: What is the task given to students?

Two liquid storage containers of the same size are being filled. Liquid enters container A at a rate of 2/3 gallon per 1/4 minute. Liquid pours into container B at a rate of 3/5 gallons per 1/6 minute.

Determine which container is being filled faster. Justify your answer. Support your answer by using evidence from the models below.
Practice analyzing classroom exhibits #1: How do you solve these problems? What do the student’s responses tell us about their understanding of these math concepts?

In the next three slides you will find three samples of student work. Determine what each response tells you about each individual student’s understanding of `CCSS.MATH.CONTENT.7.RP.A.1` and `CCSS.MATH.CONTENT.7.RP.A.2` and describe an appropriate instructional intervention that builds off the student’s strengths.

Sample Student Work 1:
Practice analyzing classroom exhibits #1: Sample Response

Check your work! A solid response would include some of the following ideas. As you read the ideas outlined below, think about what you might add/change (identify what might be missing) to make this response even stronger.

**Significant mathematical strength:**
- The student tried to calculate the unit rate by using percentages. He showed 3% of the tank filled per second. That cannot be determined, as the amount of water each tank holds was not given.

**Significant area of need:**
- The student made no connection between the amount of time and number of gallons flowing into the tanks.
- The student could not recognize a unit rate.
- The student did not make the connection between two equal representations of a ratio table.
- There was not clear understanding that a comparison had to be made comparing the rates to select which container would be filler faster.

**Intervention with Rationale:**
- I would ask/say the following: “What relationship is represented in this problem? Looking at Container A – how much water is flowing into the container and in what time period. Let’s break down the time periods and see how much water will be in the tank at each time: 1/4 hour – how much water ___________________________ A second 1/4 hour – how much water ___________________________ After a third 1/4 hour – how much water ___________________________”
- With this student, I would draw the bar model to have them fill in with time – and then the gallons that entered the tank after each 1/4 hour.
- I would ask the student how many parts of each are in 1 whole hour. Then I would ask for them to mark off that time on a Bar model – and see if they can determine the amount of water in the tank.

https://learnzillion.com/lesson_plans/2#fndtn-lesson
Practice analyzing classroom exhibits #1: How do you solve these problems? What do the student’s responses tell us about their understanding of these math concepts?

Determine what this response tells you about this individual student’s understanding of [CCSS.MATH.CONTENT.7.RP.A.1](https://example.com/ccss) and [CCSS.MATH.CONTENT.7.RP.A.2](https://example.com/ccss) and describe an appropriate instructional intervention that builds off the student’s strengths.

Sample Student Work 2:
Practice analyzing classroom exhibits #1: Sample Response

Check your work! A solid response would include some of the following ideas. As you read the ideas outlined below, think about what you might add/change (identify what might be missing) to make this response even stronger.

**Significant mathematical strength:**
- The student did understand that the filling of the containers was a linear relation – they show the water-level moving in a straight line.

**Significant area of need:**
- For Container A, the student showed a doubling of time increments. He was not consistent with his measurements.
- The container that would fill the fastest was not indicated.
- The student did not show recognition of unit rate.

**Intervention with Rationale:**
- I would first ask/say the following to get even greater information regarding this student’s understanding: “Your idea of drawing water, was an awesome idea. Can you talk to me about your model? What is it showing and how do you feel it models the situation given in the question? Can you explain your increments of time for container A? Why did you not use the same idea for container B? Do you think you could have used another type of model to help you show how much water was going into the tank for each time period?”
- Then, I would ask questions centered upon unit rates, given this is a key component involved in the ideas centered upon constants of proportionality/links to the corresponding equations and a critical area of instruction with the CCSS at the middle school level. Here, I would start by asking the student “What is a unit rate?” Based on the student’s response, I would offer an additional problem such as: How could you find the cost of one apple if 8 cost $2.40? and How could you use this unit rate to find the cost of 5 apples? Then, I would ask the student to revisit this problem, asking “What does the unit rate represent here? What would we need to find in each container to determine this unit rate? How could we use this to predict the amount of gallons present in each container after 2, 3, 4, 5, 6 minutes? How might you adjust your strategy to include the ideas regarding unit rates? and Why is that an important aspect of this problem?”
Practice analyzing classroom exhibits #1: How do you solve these problems? What do the student’s responses tell us about their understanding of these math concepts?

Determine what this response tells you about this individual student’s understanding of [CCSS.MATH.CONTENT.7.RP.A.1](https://learnzillion.com/lesson_plans/2#fndtn-lesson) and [CCSS.MATH.CONTENT.7.RP.A.2](https://learnzillion.com/lesson_plans/2#fndtn-lesson) and describe an appropriate instructional intervention that builds off the student’s strengths.

Sample Student Work 3:
Practice analyzing classroom exhibits #1: Sample Response

Check your work! A solid response would include some of the following ideas. As you read the ideas outlined below, think about what you might add/change(identify what might be missing) to make this response even stronger.

**Significant mathematical strength:**
The student has the ability to:
- Describe and identify complex fractions
- Recognize the difference between a unit rate and a ratio
- Recognize that a unite rate can be fractional
- Recognize that two equivalent ratios represent a proportion.
- Recognize the connections between the equivalent ratios and the values in the table.
- Recognize the unit rate as the constant of proportionality
- Use the constant of proportionality in an equation.

**Significant area of need:**
- The student may not have made a connection between the constant of proportionality and the unit rate, or how that is used in an equation.

**Intervention with Rationale:**
- First, I would ensure the student had a solid understanding of the connection between the constant of proportionality and how it used in an equation by asking the following questions: Let’s say, you had two equations such as $y=1/4x$ and $y=2x$. By looking at the equations only, what can you tell me about the constant of proportionality in each situation? Which has a larger constant of proportionality and how do you know? If these equations represented the two containers, without creating a table, which would fill up at a faster rate and why?

  Given this student demonstrates an understanding of identifying the unit rate and using this rate within the table, I would then provide extension questions advancing this student’s understanding of linear functions. For example, I would ask: If I were to graph the two equations given, which would be steeper and why? How do you know by looking at the equation?
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Example Analyzing Classroom Exhibits

Practice Analyzing Classroom Exhibits 2
Practice analyzing classroom exhibits #2: Additional Practice

Below you will find work samples taken from a 6th grade class. When looking at the annotations provided by the teacher, consider the four components that should be present in a level four response, note which are missing, and revise each annotation so it includes these ideas.

Work Sample (annotation begins on page 35)

Four Components:

1. Identify a specific strength that the student demonstrates - pull evidence and language from the appropriate classroom exhibits that explain why you believe this is a strength

2. Identify a specific learning need that the student demonstrates based on any discrepancies between the student’s method and response and the correct response - pull evidence and language from the appropriate classroom exhibits that explain why you believe this is an area in need of development

3. Describe the intervention that you would use to build on the student’s strength and would help support the identified area of need

4. Explain why the strategy that you choose would be effective in supporting the student in building a viable argument that is related to the standard
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Example Analyzing Classroom Exhibits

Practice Analyzing Classroom Exhibits 3
Practice analyzing classroom exhibits #3: Additional Practice

Below you will find work samples taken from a high school geometry course. When looking at the annotations provided by the teacher, consider the four components that should be present in a level four response, note which are missing, and revise each annotation so it includes these ideas.

Work Sample (annotation begins on page 9)

Four Components:

1. Identify a specific strength that the student demonstrates—pull evidence and language from the appropriate classroom exhibits that explain why you believe this is a strength.

2. Identify a specific learning need that the student demonstrates based on any discrepancies between the student’s method and response and the correct response—pull evidence and language from the appropriate classroom exhibits that explain why you believe this is an area in need of development.

3. Describe the intervention that you would use to build on the student’s strength and would help support the identified area of need.

4. Explain why the strategy that you choose would be effective in supporting the student in building a viable argument that is related to the standard.
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Example Analyzing Classroom Exhibits

Practice Analyzing Classroom Exhibits  4
Practice analyzing classroom exhibits #4: Additional Practice

Below you will find work samples taken from a 7th grade class. When looking at the student work samples, consider the four components that should be present in a level four response and draft a complete annotation for one of the student work samples.

Work Sample (pages 36-39)
Please note: Additional Work Samples are also available for practice within this document.

Four Components:

1. Identify a specific strength that the student demonstrates- pull evidence and language from the appropriate classroom exhibits that explain why you believe this is a strength

2. Identify a specific learning need that the student demonstrates based on any discrepancies between the student’s method and response and the correct response- pull evidence and language from the appropriate classroom exhibits that explain why you believe this is an area in need of development

3. Describe the intervention that you would use to build on the student’s strength and would help support the identified area of need

4. Explain why the strategy that you choose would be effective in supporting the student in building a viable argument that is related to the standard
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Example Analyzing Classroom Exhibits

Practice Analyzing Classroom Exhibits  5
Practice analyzing classroom exhibits #5: What skills and knowledge are encompassed within this standard?

A teacher is working with students in a high school algebra class. The class is currently working on the following standards:

- **Standard: 10.P.7** - Solve everyday problems that can be modeled using linear, reciprocal, quadratic, or exponential functions. Apply appropriate tabular, graphical, or symbolic methods to the solution. Include compound interest, and direct and inverse variation problems. Use technology when appropriate.

- **Standard: Mathematics.F.IF.2.04** - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

In particular, the class is working on using graphs to interpret key features. Previous classes have focused on graphing quadratic functions.
Practice analyzing classroom exhibits #5: What is the task given to students?

The graph to the right represents \( y \), the height in feet of a ball, \( x \) seconds after the ball was thrown upward from a bridge that crosses a river.

1. What is the \( y \)-intercept of the graph? Show or explain how you got your answer.

2. What does the \( y \)-intercept represent in the context of this situation?

3. After how many seconds did the ball reach its maximum height? Show or explain how you got your answer.

4. What is the maximum height, in feet, the ball reached? Show or explain how you got your answer.

5. After how many seconds did the ball reach the surface of the river? Show or explain how you got your answer.

http://www.doe.mass.edu/mcas/student/2014/question.aspx?GradeID=10&SubjectCode=mth&QuestionID=36301#
Practice analyzing classroom exhibits #5: How do you solve these problems? What do the student’s responses tell us about their understanding of these math concepts?

Below you will find a student’s response to the task given. Determine this student’s understanding of the problem as well as the standards outlined. Then draft a response using the four main components.

Sample Student Work:

A) The y-intercept of the graph is \(-\frac{1}{4}\). Starting at the highest number (top of mountain) going down to where the next point is.

B) The y-intercept represents the height in feet of the ball.

C) After 2 seconds, the ball had reached its maximum height. On the graph it shows the height on the y-axis and the time on the x-axis, leaving at the highest height and the time it reached it determines how long it took.

D) The maximum height the ball reached was 144 ft. To determine this you look at the highest point on the graph.

E) After 5 seconds the ball reached the surface of the river. Here you look at the lowest point on the graph, and the time it reached.
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- Example Analyzing Classroom Exhibits

Practice Analyzing Classroom Exhibits
Optional Practice

If you would like practice writing additional constructive responses, the following website offers multiple student work samples. For each task, you can view on-target responses as well as responses that require additional support. To use the links below, click on the grade level and then select the domain (Algebra, Geometry, etc.) /type of question you would like to practice. You will then see the task with the corresponding rubric. Click on the level of student understanding within the rubric to see sample student responses.

- Grade 7
- Grade 8
- Grade 10